# MATH–305 Fixed Point Method with Aitken Acceleration Dr. TeBeest

In a previous assignment, we saw that the equation

$$e^x \; = \; 2x^2 \tag{1}$$

has three solutions on the interval $-2 \le x \le 3$.

1. Most equations can be expressed in fixed point form $(x = g(x))$ in multiple ways. For example, show that Eq. (1) can be expressed as

$$x \; = \; \pm\sqrt{\frac{e^x}{2}}\,. \tag{2}$$

2. To approximate the negative root, we must use the negative sign in Eq. (2).

   (a) Plot functions $x$ and $-\sqrt{e^x/2}$ on a common plot in Maple on the interval $-1 \le x \le 0$. Restrict the vertical range to $-1 \le y \le 0$, make the curves respectively red and blue, give them a thickness of 4 or 5, make the plot size $300\times300$ pixels, and include the plot option `scaling = constrained`.

   (b) Using $x_0 = -1.0$, apply 4 iterations of the fixed point method **with Aitken acceleration** to approximate the root near $x = -0.5$.

3. Eq. (1) can be expressed in another fixed point form by solving for the $x$ in the exponent. Do so to show that we obtain

$$x \; = \; \ln\left(2x^2\right). \tag{3}$$

   (a) Plot functions $x$ and $\ln\left(2x^2\right)$ on a common plot in Maple on the interval $0 \le x \le 4$. Restrict the vertical range to $0 \le y \le 4$, make the curves respectively red and blue, give them a thickness of 4 or 5, make the plot size $300\times300$ pixels, and include the plot option `scaling = constrained`.

   (b) Using $x_0 = 4.0$, apply 4 iterations of the fixed point method **with Aitken acceleration** to approximate the root near $x = 2.5$.