Newton–Cotes Integration Formulas

The approximation of a proper, definite integral of function $f(x)$ on each section $[x_0, x_d]$ using Newton-Gregory (N-G) polynomials of degree d is given by

$$I_1 \;=\; \int_{x_0}^{x_d} f(x)\,dx \;\approx\; \frac{d}{c}\,h\left[\,a_0 f_0 + a_1 f_1 + \cdots + a_d f_d\,\right] \;=\; \frac{d}{c}\,h\sum_{k=0}^{d} a_k f_k\,, \qquad (1)$$

where d is the degree of interpolating polynomial on each section, $h = (b-a)/n$ is the abscissa spacing, and the other constants used in (1) are given by

d	c	a_0	a_1	a_2	a_3	a_4	a_5	a_6	a_7	Global Error	D
1	2	1	1							$O(h^2)$	1
2	6	1	4	1						$O(h^4)$	3
3	8	1	3	3	1					$O(h^4)$	3
4	90	7	32	12	32	7				$O(h^6)$	5
5	288	19	75	50	50	75	19			$O(h^6)$	5
6	840	41	216	27	272	27	216	41		$O(h^8)$	7
7	17280	751	3577	1323	2989	2989	1323	3577	751	$O(h^8)$	7

- A dth degree N-G polynomial will integrate all polynomials up to degree D exactly.

  For example, the 4th degree N-G polynomial integrates all polynomials up to degree 5 *exactly*.

- The number of subintervals n dividing interval $[a, b]$ must be divisible by d.

  For example, in the $d = 4$ case, the number of subintervals n must be divisible by 4.

- Each section comprises d successive subintervals.

  For example, in Simpson's–1/3 rule $(d = 2)$ there are $n/2$ sections:

  Section 1: subintervals 1 & $2 \rightarrow [x_0, x_2]$
  Section 2: subintervals 3 & $4 \rightarrow [x_2, x_4]$
  Section 3: subintervals 5 & $6 \rightarrow [x_4, x_6]$
  ...
  Section $n/2$: subintervals $(n-1)$ & $n \rightarrow [x_{n-2}, x_n]$

- In each formula, the a coefficients add to c: $\sum a_k = c$.

- The even degree N-G polynomials are *very slightly* more accurate than the next higher odd degree formulas. For example, although both have $O(h^4)$ accuracy, Simpson's–1/3 rule $(d = 2)$ is *very slightly* more accurate than Simpson's–3/8 rule $(d = 3)$.